Throughout it has been assumed that the surface of the liquid film is flat and that the edges of the film do not significantly effect the exposed area. The ratio of belt width to film thickness in all runs was in the range of 25 to 125. To test the uniformity of the film a movable probe constructed from a hypodermic needle was inserted into the chamber. The tip of the probe touched the moving belt. When viewed through a traveling microscope a pattern of ripples could be seen above the tip of the probe. The probe was moved across the belt, and the resulting ripple pattern gave a qualitative indication of the film thickness across the belt.

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NOTATION

$\bar{C}$ = average liquid concentration, moles/cc.
C_o = initial liquid concentration, moles/cc.
C^* = equilibrium liquid concentration, moles/cc.
δ = film thickness, cm.
D = diffusivity, sq.cm./sec.
k_s = interfacial mass transfer coefficient, cm./sec.
L = exposed belt length, cm.
Q = volumetric flow rate, cc./sec.
t = contact time, sec.
u = velocity, cm./sec.
w = belt width, cm.
x = distance normal to interface, cm.

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Frictional Pressure Drop in Two-Phase Flow: A. A Comparison of Existing Correlations for Pressure Loss and Holdup

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The current status of knowledge and practice in two-phase flow has been well summarized in the following paragraph (*14*):

"Multiphase flow still suffers when compared on a theoretical basis with other general fields of flow theory as is natural in view of the great complexity of the problem. We may note, however, that the buildup of empirical knowledge in this field is now very impressive. *What is most needed are critical reviews of the various specialized subjects such as pressure drop* in horizontal and vertical pipes, flow regimes, atomization, and the mechanics of fluid particles."

The total number of experimental measurements of two-phase pressure drop is currently well over 20,000, half of which have been obtained since 1959. It is evident that two-phase flow is a timely subject. Furthermore this continued accumulation of data demonstrates that there is not yet even a phenomenological understanding of this type of flow.

The purpose of the first part of this paper is to critically compare the prediction of certain correlations for pressure drop and holdup in horizontal flow with selected experimental data. The second part presents a new analysis of frictional energy loss.

DATA BANK

The first problem one must solve in a comprehensive study using information from various sources is that of data handling. In order to characterize a single two-phase flow measurement it is necessary to specify two flow rates, five fluid properties, a pipe diameter, several measurements of length, a test section orientation as well as an observed flow pattern, and measurements of holdup, where made. Furthermore there are three or more units in common usage for each quantity. Since there are tens of thousands of data points to be dealt with, it becomes apparent that use of an automatic computer is virtually a necessity.

Original source data were collected from theses, ADI reports, and directly from experimental run sheets, in some cases. In numerous instances investigators provided detailed run data. Approximately 15,000 data points were collected in this way from both published and unpublished sources. A rather flexible coding system was developed enabling data to be put on punched cards in units "as received". A program was then written to prepare a deck of IBM cards, where all data appear with a common set of units and in floating point notation. This deck, designated as the *data bank*, is in a format permitting rapid location of data covering any specified range of conditions. The data can be grouped according to the run conditions, test section characteristics, or experimental results, as desired. At present the data bank contains approximately 9,000 data points. It is available at cost to interested investigators. As described below, this data bank was used to test certain published correlations.

STATISTICAL PARAMETERS

The following procedure was used to test both the pressure drop and holdup correlations under study:

1. The operating variables for each data point i were used in conjunction with the correlation being tested to generate a predicted value P_i of either pressure drop or holdup for that point.

2. The predicted value was compared with the measured value M_i for the point by calculating a fractional deviation d_i defined as

$$d_i = \frac{P_i - M_i}{M_i} \tag{1}$$

3. The arithmetic mean deviation for n points is then

$$\overline{d} = \left(\sum_{i=1}^{n} d_i \right) \Big/ n \tag{2}$$

and the so-called *standard deviation* is

$$\sigma = \frac{\sqrt{\sum_{i=1}^{n} (d_i - \overline{d})^2}}{n - 1} \tag{3}$$

The standard deviation is a completely descriptive measure of the spread between predicted and measured values only if the population of points is normally distributed. Then 68% of the population is within one σ of the mean.

The population d_i has, by virtue of its definition, a possible range of from −1.00 to infinity. Because of this fixed lower bound, d_i cannot be normally distributed. Furthermore most correlations based on experimental data are biased toward certain ranges of operating conditions so that fractional deviations are not normally distributed. In order to provide a better measure of the spread, a variable ψ was defined as the fractional deviation above and below the mean which actually included 68% of the data points. The value of ψ was obtained by a computer count of the calculated deviations.

DATA USED TO TEST CORRELATIONS (CULLED DATA)

Any comparative evaluation of correlations which is to be definitive must compare the ability of these correlations to predict the pressure drop or holdup for identical data. In the past, claims of superiority of one correlation over another could usually be reversed simply by making the comparison for different data. It is easy to see why this is so when, for apparently similar test conditions, pressure drop data of different investigators vary by 30 to 60%. Thus the definitive comparison of correlations is not possible until definitive tests of the data itself are made and those data which are not reliable discarded. It was thus necessary to cull the 15,000 data points in the bank to obtain the data with a high degree of reliability.

Owing to errors of various kinds, all experimental data suffer from shortcomings in precision and accuracy. This is true for two-phase flow data in particular, it seems. The main sources of experimental error in the data are pressure drop measurement, where the lead lines must be maintained in a single-phase condition, and where pulsating pressures must be damped; and holdup measurements at low R_L conditions. The extent of these errors of course varies from one investigation to the next, and in some instances they are very large indeed.

For pressure drop data, in order to find blunder-type (recording and typographical for example) errors, graphs were prepared showing all the data of a given investigator in the form $-dP/dL$ vs. the two flow rates W_L and W_G. Certain points were discovered which obviously could not be consistent with the trends shown by neighboring points. These points, which were discarded, amounted to 5% of the data under consideration.

The graphs also showed a great deal more scatter in regions where the measured pressure drop was low. In each case the experimental technique was evaluated to obtain an estimate of the measuring accuracy. All data reporting pressure drops less than ten times this value were discarded. This criterion eliminated 7% of the points under consideration.

Data points were rejected where flow rate was determined by means of phase calculations rather than by direct measurement to eliminate any possible errors due to questionable equilibrium data.

For the purpose of this study, data from nonhorizontal test sections were eliminated. In addition those obtained in very short test sections where entrance effects are known to be large were discarded.

After this culling procedure approximately 5,000 data points remained. The surviving points were then studied carefully. Certain data were eliminated where bends or fittings in the test equipment caused results which were unique to the equipment used. Where data from two investigators covered the same range of conditions, and predicted about the same results, the data with significantly wider scatter was eliminated. Finally some data were discarded because of heavy duplication for certain pipe sizes and flow ranges. Indeed if all the data had been used, the results would have been biased toward the small pipe sizes since much more data have been taken in this region.

The 2,620 points which were left comprised the culled data. This is at least ten times the number used in establishing any of the correlations tested. In only one case (Chenoweth-Martin data) does the culled data include some points used in the original establishment of the correlations tested. The data were selected as being representative of other data in their class, covering a wide range of pipe diameter and system properties, and having a relatively small internal scatter.

These culled data are described in Table 1. For the two-component systems the approximate pipe sizes are shown to cover the range of 1 to 5½ in., and the approximate liquid viscosities 1 to 20 centipoise. The one-compo-

TABLE 1. DESCRIPTION OF THE CULLED DATA

A. PRESSURE GRADIENT

(1) Two-Component Flow

Pipe Diam., In.	Number of Data Points For Liquid Viscosity (CP) of 1	3	20
1	224 (18)*	230	156
2	320 (7)	398 (7)	401 (7)
3½	109 (3)	67	111
5½	24 (16)	131	122

(2) One-Component Flow

Pipe Diam., In.	Number of Data Points For Pressure (PSIA) Range of 0 – 100	400 – 800	1000 – 1400
½	–	103 (17)	131 (12)
1	93 (13)	–	–

B. HOLD UP

Pipe Diam. In.	Number of Data Points For Liquid Viscosity (CP) of 1	3	20
1	60 (6)	83	78
2	–	55 (7)	103 (7)
3½	–	29	90
5½	–	104	104

* Numbers in Brackets Give Reference for Source of Data

nent flow data is classified into two pipe sizes and three pressure ranges.

INTERNAL SCATTER OF CULLED DATA

The culled data contains sets of relatively consistent observations. However even these best data reflect errors in measurement and exhibit internal scatter. If a set of data shows a characteristic scatter of say 15%, one could not expect even a perfect correlation to predict this set with a scatter of less than 15%. The value of σ, that is the scatter for a correlation, must be compared with the internal scatter in the data itself.

An attempt was made to determine the scatter of each set of culled data by use of regression analysis. For each data set in Table 1 (for example, 224 data points for 1-in. diameter and 1 C_P liquid viscosity) the only independent variables of any consequence are liquid and gas flow rates, with measured pressure gradient as the dependent variable. Regression analysis can then provide the best three-dimensional smooth surface through the region of the measurements, subject to the following polynomial form of equation:

$$\left(\frac{\Delta P}{\Delta L}\right) = \sum_{i=0}^{n} \sum_{j=0}^{i} \alpha_{i-j,j} W_L^{i-j} Q_G^{j} \qquad (4)$$

Forms through sixth order ($N = 6$, with 28 coefficients) were generated. The equations were then used to calculate the pressure drop at each of the given rate pairs. Calculated and measured pressure drops were then compared statistically using the parameters defined by Equations (1) to (3). For the sake of clarity call these $\bar{d}_D$, σ_D, and ψ_D.

As one would expect, the use of a higher-order equation gave decreasing σ_D and ψ_D ($\bar{d}_D$ was close to zero for all orders) as the order was increased from zero to one, one to two, two to three, and in some cases three to four. In all cases σ_D and ψ_D leveled off as the higher orders of five and six were reached.

The implication is that these high-order values of σ_D and ψ_D characterize experimental errors inherent in the data themselves, and on the same set of observations even a perfect pressure drop correlation could be expected to perform no better. These calculated values are given in the last column of Table 2a. The 3½ and 5½ in. air-water values could not be obtained owing to an insufficient number* of observed experimental points. For the same reason the steam data in Table 2b could not be analyzed.

* Although 109 measurements are shown for the 3-½ in. air-water data, these were distributed into several ranges of system pressure; in each of which were too few points to work with.

TABLE 2a. TEST OF PRESSURE DROP CORRELATIONS†, TWO-COMPONENT SYSTEMS, EFFECTS OF LINE SIZE AND LIQUID VISCOSITY

D, in	μ_L CP	BAKER $\bar{d}$	BAKER σ	BAKER ψ	BANKOFF $\bar{d}$	BANKOFF σ	BANKOFF ψ	CHENOWETH MARTIN $\bar{d}$	CHENOWETH MARTIN σ	CHENOWETH MARTIN ψ	LOCKHART MARTINELLI $\bar{d}$	LOCKHART MARTINELLI σ	LOCKHART MARTINELLI ψ	YAGI $\bar{d}$	YAGI σ	YAGI ψ	n	σ_D	ψ_D
1	1	64.2	40.0	45.0	2080	980	–	– 8.5	17.8	15.0	– 6.6	10.1	10.0*	40.9	29.1	30.0	224	6.0	5.0
	3	77.4	335	87.5	1172	2220	–	11.2	55.6	30.0	3.8	29.1	20.0*	183	123	–	230	22.0	17.0
	20	30.7	89.5	40.0	737	1384	–	42.5	94.2	65.0	– 5.5	24.7	20.0*	481	192	–	156	18.0	14.0
2	1	–13.6	60.3	65.0	1178	2910	–	– 2.7	24.8	20.0*	9.2	37.7	25.0	162	228	–	320	16.0	7.0
	3	19.3	79.0	82.5	4810	4654	–	8.4	45.3	45.0	– 4.7	22.9	25.0*	62.3	74.5	80.0	398	16.0	9.0
	20	73.0	159	90.0	2804	4893	–	95.4	268	–	13.2	52.9	30.0*	271	325	–	401	24.0	8.0
3½	1	11.5	79.2	82.5	2176	3072	–	15.0	40.2	30.0*	31.0	50.2	47.5	27.6	104	97.5	109	–	–
	3	7.1	60.1	72.5	4720	5000	–	27.8	62.0	45.0	16.3	39.3	22.5*	84.5	86.1	–	67	18.0	12.0
	20	31.8	60.6	47.5	2432	3561	–	51.0	91.8	57.5	– .4	26.2	22.5*	147	83.4	–	111	27.0	13.0
5½	1	–70.5	11.6	10.0	254	213	–	51.2	23.7	30.0	38.3	12.2	12.5*	93.3	22.3	22.5	24	–	–
	3	– .5	44.6	45.0	2096	3704	–	20.0	57.5	45.0	11.6	41.5	37.5*	106	80.8	–	131	19.0	13.0
	20	9.6	47.8	50.0	2692	5263	–	37.1	79.4	47.5	– 1.0	24.8	25.0*	120	69.0	–	122	21.0	7.0
ALL DATA POINTS		28.2	159	65.0				27.6	89.5	42.5	4.2	36.0	25.0*	155	184		2293		

† $\bar{d}$, σ, and Ψ expressed as percentages.

* Asterisks in this and the following tables show which correlation seemed to predict best for each row.

TABLE 2*b*. TEST OF PRESSURE DROP CORRELATIONS, ONE-COMPONENT SYSTEM

Pressure Range PSIA	D In.	BAKER $\bar{d}$	BAKER σ	BAKER ψ	BANKOFF $\bar{d}$	BANKOFF σ	BANKOFF ψ	CHENOWETH MARTIN $\bar{d}$	CHENOWETH MARTIN σ	CHENOWETH MARTIN ψ	LOCKHART MARTINELLI $\bar{d}$	LOCKHART MARTINELLI σ	LOCKHART MARTINELLI ψ	YAGI $\bar{d}$	YAGI σ	YAGI ψ	n
25–100	1	234	238	–	4,852	3,964	–	53.5	85.9	67.5	39.5	66.6	55.0	−38.8	65.2	45.0*	93
400–800	½	632	368	–	377	206	–	55.9	88.2	65.0	63.6	57.0	52.5*	−87.7	8.8	7.5	103
1000–1400	½	358	177	–	31.9	73.0	52.5	63.6	145	112	−17.5	21.1	17.5*	−85.6	16.2	10.0	131
ALL DATA POINTS		414	310					6.6	128		23.5	60.7	*	−72.6	42.6		327

TEST OF PRESSURE DROP CORRELATIONS

Approximately twenty-five correlations for pressure drop have appeared in the literature. Five of these, which have found rather wide use in two-phase flow literature, are Baker (*1*), Bankoff (*2*), Chenoweth and Martin (*3*), Lockhart and Martinelli (*10*), and Yagi (*19*).

Each of these correlations was tested with the culled data. Details of the correlations can be found in the original papers, and the equations and curve fits used for computer programming can be found in reference *4*. All five of the correlations contain constants which were evaluated from experimental data. In some cases these constants were calculated from data representing a narrow range of conditions. In others the constants represented smoothed values obtained for a broad spectrum of data. In the case of Bankoff's correlation his empirical constant was derived from steam-water data. For each correlation it is possible to find a value of the constant or constants which will give exact agreement with any single experimental point. However, the constant varies in magnitude from condition to condition, and predicting the experimental constant becomes a task almost as formidable as evolving the correlation in the first place. In this study no attempt was made to modify the constants and each correlation was tested over the full range of the culled data with the constants evaluated as the author suggested. It is possible, then, that each of these correlations may be improved for each set of data by adjustments in the value of the constants.

Two Component Systems

Table 2*a* summarizes the results of pressure drop correlation tests on two-component systems. Examination of this table shows, first of all, that the Bankoff and Yagi correlations are entirely inadequate for horizontal two-phase flow of two-component systems. The table shows an almost uniform trend for the correlations of Chenoweth-Martin and Lockhart-Martinelli: deviations become larger as pipe diameter increases. On the other hand some natural gas-crude oil data taken in large diameter lines were used to establish the Baker correlation, and it seems to perform better for large pipe sizes and more viscous liquids.

Analysis of the table by rows reveals the Lockhart-Martinelli correlation to be better than any of the others in all except two cases, where the Chenoweth-Martin correlation is best: 2 and 3½ in. air-1CP liquid (water). This correlation was developed from data on an air-water system in 1½ and 3-in. pipes. The Chenoweth-Martin correlation shows a clear trend of progressively poorer agreement with measurements as liquid viscosity increases for each given pipe size.

An overall comparison of the five correlations tested with all the two-component flow data is shown at the bottom of Table 2*a*. The Martinelli correlation is clearly better than the other four tested.

Table 3 shows a breakdown by flow pattern. Except in one case, plug flow, where the Chenoweth-Martin correlation is best, the Lockhart-Martinelli correlation is better than the others.

TABLE 3. TEST OF PRESSURE DROP CORRELATIONS, EFFECT OF FLOW PATTERN

OBSERVED FLOW PATTERN	BAKER $\bar{d}$	BAKER σ	BAKER ψ	BAKER n	CHENOWETH MARTIN $\bar{d}$	CHENOWETH MARTIN σ	CHENOWETH MARTIN ψ	LOCKHART MARTINELLI $\bar{d}$	LOCKHART MARTINELLI σ	LOCKHART MARTINELLI ψ	n†
Bubble	15.6	168	30.0	960	–	–	–	–	–	–	0
Plug	116	92.9	100	69	− 6.5	19.8	10.0*	9.4	36.3	20.0	270
Stratified	–	–	–	–	122	103	–	23.3	33.0	22.5*	34
Wave	−91.0	2.5	2.5	98	138	301	135	38.4	85.7	42.5*	287
Slug	61.0	218	135	251	12.8	92.1	27.5	2.9	31.2	17.5*	974
Annular	68.7	81.2	100	430	22.2	81.1	65.0	−12.8	35.6	30.0*	265
Dispersed	16.9	35.0	30.0	133	84.4	81.1	85.0	18.0	34.1	25.0*	111

† Except for Baker's Correlation, which is based on the number of points calculated (4) to be in each flow regime, all correlations are tested on the same points.

One-Component System

Table 2*b* summarizes the results of pressure drop correlation tests on one-component systems. The Bankoff and Yagi correlations are better here than for the two-component data tested. It appears that Bankoff's correlation improves with increasing pressure. At the highest pressure nevertheless the Lockhart-Martinelli correlation is best. At the lowest pressure Yagi's correlation is best.

The overall comparison shows, as in two-component flow the Martinelli correlation is better than the other four tested. The expected error for one-component flow is much greater than for two-component flow however.

DISCUSSION

The total measured pressure drop in two-phase flow consists of three contributions:

$$\left(\frac{\Delta P}{\Delta L}\right)_{\text{Measured}} = \left(\frac{\Delta P}{\Delta L}\right)_{\text{Friction}} + \left(\frac{\Delta P}{\Delta L}\right)_{\text{Acceleration}} + \left(\frac{\Delta P}{\Delta L}\right)_{\text{Elevation}} \tag{5}$$

For flow in a horizontal line of course the last term is zero. The importance of the acceleration term is responsible in large measure for the apparent difficulty which has existed for 20 yrs. in correlating pressure drop measurements from different equipment.

The manner in which the acceleration term should be determined depends on the flow pattern or, more to point, the distribution of liquid and gas over the pipe cross section. In a near homogeneous flow for example the velocity is nearly uniform, and a mixture velocity change is used:

$$(\Delta P)_A = \frac{1}{g_c A_p}\left[\begin{pmatrix}\text{Momentum}\\ \text{flow rate}\\ \text{downstream}\end{pmatrix} - \begin{pmatrix}\text{Momentum}\\ \text{flow rate}\\ \text{upstream}\end{pmatrix}\right] \tag{6}$$

On the other hand, in stratified or wave or annular flow where considerable slip exists

$$(\Delta P)_A \equiv (\Delta P)_{AL} + (\Delta P)_{AG} \tag{7}$$

$$(\Delta P)_{AL} = \frac{W_L}{g_c A_p}(V_{L2} - V_{L1}) \tag{8}$$

$$(\Delta P)_{AG} = \frac{W_G}{g_c A_p}(V_{G2} - V_{G1}) \tag{9}$$

or, if the liquid velocity varies across the conduit

$$(\Delta P)_{AL} = \frac{1}{g_c A_p}\Sigma_j G_{Lj}\,\Delta V_{Lj}\,\delta A_j \tag{10}$$

where G_{Lj} is the liquid mass flux through area δA_j, and $\Sigma_j \delta A_j = A_p$.

In the absence of such detailed data on the phase distributions the axial change in holdup may be as an approximation:

$$\left(\frac{\Delta P}{\Delta L}\right)_{AL} = \frac{G_L^2}{g_c \rho_L}\frac{\Delta}{\Delta L}\left(\frac{1}{R_L}\right) \tag{11}$$

Table 4. Test of Holdup Correlations, Effect of Range of R_L

Range of measured R_L	Hoogendoorn $\bar{d}$	Hoogendoorn σ	Hoogendoorn ψ	Hughmark $\bar{d}$	Hughmark σ	Hughmark ψ	Martinelli $\bar{d}$	Martinelli σ	Martinelli ψ	n
.80 – .99	– 1.8	10.0	5.0*	– 3.0	10.0	6.0	2.0	13.0	10.0	193
.60 – .79	– .05	11.0	10.0	– 6.1	13.0	12.5	0.9	9.3	10.0*	111
.40 – .59	2.9	18.0	17.5*	–11.0	20.0	20.0	8.5	20.0	17.5	124
.20 – .39	0.5	33.0	27.5	–11.0	22.0	22.5*	23.0	33.0	35.0	145
.10 – .19	18.0	74.0	25.0	20.0	44.0	27.5*	51.0	72.0	57.5	74
.06 – .099	69.0	72.0	57.5	69.0	41.0	32.5*	57.0	71.0	42.5	34
.01 – .059	295	208	–	303	262	–	276	377	–	40

$$\left(\frac{\Delta P}{\Delta L}\right)_{AG} = \frac{G_G^2}{g_c}\frac{\Delta}{\Delta L}\left(\frac{1}{\rho_G(1-R_L)}\right) \tag{12}$$

Both holdup and gas density depend on total pressure. Since pressure changes along the axis of flow, R_L and ρ_G change, and their rates of change per unit length of pipe establish the pressure gradient terms on the left sides of the above equations. These add, in accordance with Equation (5), to the frictional pressure gradient to give the total pressure gradient, but it is this total pressure gradient on which the magnitude of the acceleration term itself depends. If the total pressure gradient is large, the effects of acceleration will be large also.

Indeed Magiros and Dukler (*11*) have shown that in small lines, where the pressure gradients are characteristically much larger than in large lines, 50% of an experimentally measured pressure drop may be due to the fact that the gas expands, causing both the liquid and the gas to accelerate. Correlations developed from data in small pipes where acceleration effects are important cannot be expected to predict adequately total pressure loss in large diameter pipes unless the acceleration term is accounted for, or unless both frictional and acceleration terms depend on pipe size in the same manner.

A reliable holdup correlation involving ρ_G may be used to estimate the acceleration term in a design case. If only one measurement of R_L is made in an experiment, the acceleration term is not calculable; the kind of data needed is the type obtained by Isbin (9) and some other investigators, (*5*), (*15*), where R_L is reported at various axial positions.

Each of the five pressure drop correlations tested was developed from experimental data on total measured pressure drop. The comparisons of prediction with data were therefore necessarily made on this same basis. In the light of the importance of acceleration terms the reason for certain trends shown in Table 2*a* becomes apparent.

The progressively larger deviation shown by the Lockhart-Martinelli correlation as pipe diameter increases is probably due to its establishment from data taken for flow in small pipes where the acceleration terms contribute greatly.

It should be noted that each of the five correlations tested depend on experimental data, at least in part, to provide either certain constants or correlating curve locations. In every case the constants or the curves recommended by the author were used in the test of his own correlation. However all correlations were tested with the same data. It seems likely that the agreement between prediction and data for some of these correlations can be improved if the constants or curves are re-evaluated to force a fit with the culled data. In most cases the correlations were constructed with data covering limited ranges of conditions. The task of finding experimentally dependent factors for the range of the culled data is a much greater one.

HOLDUP CORRELATIONS

The following holdup correlations have been tested: Hoogendoorn (*7*), Hughmark (*8*), and Lockhart-Martinelli (*10*). It should be noted that the Hughmark correlation is a modification of one originally proposed by Bankoff (*2*).

For about one-third of the data points used in checking the pressure drop correlations, simultaneous measurements of holdup were reported also. The above three holdup correlations were first tested with all of the data. These results showed extremely poor performance by all the correlations.

The difficulty of making accurate measurements of small values of holdup has been noted above. Holdup measure-

TABLE 5. TEST OF HOLDUP CORRELATIONS, EFFECTS OF LINE SIZE AND LIQUID VISCOSITY

D, in	μ_L CP	HOOGENDOORN $\bar{d}$	HOOGENDOORN σ	HOOGENDOORN ψ	HUGHMARK $\bar{d}$	HUGHMARK σ	HUGHMARK ψ	MARTINELLI $\bar{d}$	MARTINELLI σ	MARTINELLI ψ	n
1	1	71.8	62.7	47.5	13.8	25.7	25.5	− 1.8	27.6	32.5*	60
	3	6.9	21.1	15.0	0.7	16.1	12.5*	18.7	30.7	22.5	83
	20	−20.0	23.2	15.0	−20.3	15.3	15.0	− .4	16.8	12.5*	78
2	3	0.4	22.5	17.5*	− 6.7	25.8	14.0	1.1	36.3	17.5	55
	20	9.7	62.3	12.5	1.0	35.4	17.5*	22.0	49.5	27.5	103
3½	3	11.9	15.6	17.5	− .3	16.9	17.5*	6.2	26.7	22.5	29
	20	13.3	19.7	12.5	1.9	24.8	22.5*	25.3	43.1	32.5	90
5½	3	2.6	18.9	10.0*	− 3.2	19.8	15.0	14.7	31.7	17.5	104
	20	− 3.6	11.9	7.5*	− 6.2	13.1	11.5	7.4	16.0	15.0	104
ALL DATA POINTS		8.2	40.5	17.5	− 2.5	24.1	17.5	12.0	34.6	22.5	706

ment problems have been reviewed by Isbin (9), among others (5), (15). It is generally agreed that no matter which experimental method is used, the per cent error becomes larger as R_L goes down. Table 4 shows how the correlations agree with various ranges of measured R_L. For each correlation there is a practically uniform trend in both $\bar{d}$ and σ, becoming worse for the lower ranges and indeed meaningless for the lowest, of 0.01 to 0.059. Here, as the table shows, either the measured value is consistently too small or the calculated too large. Considering the experimental difficulties inherent in this range, one is led to question the validity of the data itself, rather than that of the correlations. All the data in Table 4 were taken by Hoogendoorn (7) using a capacitance technique which was calibrated by three different methods. He found that the techniques agreed to within a standard deviation of 0.03 absolute units in R_L.

A fairer test of the correlations then was made with the more reliable data, where measured values of R_L were one tenth or greater. At this level of R_L the standard deviation of 0.03 would give an average inherent error due to measurement of 33%. These 706 data points constitute the culled data for holdup comparison and are classified in Table 1. The results of the comparisons are given in Table 5 showing pipe diameter and liquid viscosity effects. Trends in this case are more subtle than those for pressure drop in Table 2. In every case except Hoogendoorn's predictions for air water in a 1-in. line, the standard deviation σ is greatest for air-20 centipoise oil flowing in a 2-in. line. Consideration of the overall results shown at the bottom of the table shows Hughmark's correlation to be clearly better than the others tested.

Table 6 regroups the data of Table 5 according to observed flow pattern (the 1-in. air-water data is not included). In four of the six cases the Hughmark correlation is best.

SUMMARY AND CONCLUSIONS

The Lockhart-Martinelli correlation, oldest of the five tested, shows the best agreement with a set of carefully culled experimental data on pressure drop. Comparison of scatters between this correlation and the data and between the regression analysis and the data indicates that the correlation is far from perfect. Hughmark's correlation for holdup gives the best agreement with holdup data.

TABLE 6. TEST OF HOLDUP CORRELATIONS, EFFECT OF FLOW PATTERN

OBSERVED FLOW PATTERN	HOOGENDOORN $\bar{d}$	HOOGENDOORN σ	HOOGENDOORN ψ	HUGHMARK $\bar{d}$	HUGHMARK σ	HUGHMARK ψ	MARTINELLI $\bar{d}$	MARTINELLI σ	MARTINELLI ψ	n
Plug	3.6	8.9	7.5	− 3.5	7.3	5.5*	1.0	7.4	7.5	113
Stratified	22.4	28.1	22.5	−27.7	15.8	12.5*	− 1.5	23.8	22.5	18
Wave	0.6	23.3	22.5*	−14.3	21.4	22.5	4.2	29.2	25.0	68
Slug	2.7	35.9	12.5	− 1.0	25.2	15.0*	20.2	38.9	25.0	412
Annular	−43.2	34.3	31.0	−20.4	34.6	29.0	1.0	38.4	35.0*	13
Dispersed	2.6	31.9	32.5	− 2.6	25.3	20.0*	− 4.0	29.5	25.0	22

NOTATION

- A = cross-sectional area, sq. ft.
- d = fractional deviation between calculated and measured values
- $\bar{d}$ = arithmetic mean deviation
- D = pipe diameter, ft.
- G = mass velocity, lb.$_m$/(sec. sq. ft.)
- g_c = conversion factor, 32.174 lb.$_m$ ft./(lb.$_f$ sec.^{2})
- L = length of pipe, ft.
- n = total number of data points in a given set
- P = pressure, lb.$_f$/(sq. ft.)
- Q = volumetric flow rate at average conditions, cu. ft./sec.
- R_L = holdup or fraction liquid volume in the conduit under actual conditions
- V = time average axial velocity, ft./sec.
- W = mass flow rate, lb.$_m$/sec.
- ρ = density, lb.$_m$/cu. ft.
- σ = standard deviation
- ψ = fractional deviation which includes 68% of a population

Subscripts

- A = acceleration
- D = data
- G = gas phase
- L = liquid phase
- P = pipe
- 1, 2 = upstream and downstream conditions, respectively

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